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Engineering Notes

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Comparative Masses and Energies for Lunar and Martian Missions

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I. Introduction

THE use of any staged sequence of rockets implies a fixed cost in energy and initial mass relative to the mass of payload delivered. With each burn phase, the cumulative mass investment grows exponentially because the final mass at each phase of a mission is weighted by the fuel mass required to provide the kinetic energy change needed in that phase.

A similar accumulation is seen for the energy invested in any sequence of maneuvers. Starting from the final destination, one can then calculate the specific mass and invested energy of each stage, working backward to the initial mass and energy requirements. Here we focus on those requirements at low earth orbit (LEO) for several straightforward mission profiles to the moon and to Mars.

The energy investment to obtain LEO is also considerable, for whereas the final orbital speed at LEO is about 7.73 km/s, the equivalent energy investment to LEO requires a δV of about 9.124 km/s (including work done against gravity and drag) (private communication, R. Anacker). When the formulation developed here is applied, with a booster specific impulse (I_{sp}) of 455 s, a payload of 1 kg delivered to LEO requires 7.80 kg at rest for launch and represents about 93.62 MJ of invested energy.

II. Formulation

We first discuss the rules used to characterize the mechanics of each stage or distinct maneuver, followed by a description of the particular mission profiles examined. Here we use standard notation for the various mechanical quantities and MKS units throughout the development. Velocity changes are denoted δV, work done is $\mathcal{W}$, and $\dot{M}$ is the exhausted mass flow rate of a rocket stage.

Mass and Energy Within a Stage

To study the tradeoffs involved in any set of distinct mission profiles, there are two required rules. As a consequence of the well-known rocket equation, the mass sequence is given by

$$m_k = [m_{d,k} + m_{k-1}\exp[\delta V_k / I_{\mathrm{sp}} g]] \tag{1}$$

and clearly the cumulative vehicle mass grows exponentially with δV. Here m_{k-1} is the net mass of the preceding term, and the added dead mass at any point $m_{d,k}$ may be zero but usually is going to be a fraction of the needed reaction mass to boost m_{k-1}. The added mass for "tankage" includes all propulsion system elements, as well as the fuel tanks required to contain the fuel mass. In a similar way, the added mass fraction for an aeroshell includes a proportionate share of the final parachute assembly.

We examine each mission by setting the first mass in the sequence as unity, namely, $m_1 = 1$ kg. The final relative mass in the sequence is the full mass penalty per landed payload kilogram as projected back to LEO. It is the primary comparative figure of merit among mission profiles.

Next, the work done to accelerate or decelerate the craft over a given δV_k, namely,

$$\int V \cdot F \, \mathrm{d}t$$

represents the energy invested in that particular burn. For a free space burn, appropriate outside LEO, $F = \dot{M} V_{\mathrm{exh}}$ and the required integration becomes

$$\mathcal{W} = -(gI_{\mathrm{sp}})^2 \int_{t_0}^{t_f} \mathrm{d}t_1 \dot{M}\, \ln\left(\frac{M_0}{M}\right) \tag{2}$$

with $V_{\mathrm{exh}} = gI_{\mathrm{sp}}$ assumed constant. The time integration can be eliminated, and there obtains a simple quadrature,

$$\mathcal{W} = M_0(gI_{\mathrm{sp}})^2 \int_1^{\eta_f} \mathrm{d}\eta \frac{\ln(\eta_f)}{\eta_{f^2}} \tag{3}$$

based on the mass change alone, that is, invariant for any particular history of mass flow during the burn. Hence, for any particular burn,

$$\eta_f = m_k/m_{k-1} \equiv \exp(\delta V_k / I_{\mathrm{sp}} g) \tag{4}$$

and the energy requirement for the sequence of interest is given by

$$\mathcal{W}_k = \left(m_{k-1} V_{\mathrm{exh}}^2 / 2\right)\{\eta_f - [1 + \ln(\eta_f)]\}2 \tag{5}$$

Now the sum of all such energy changes over the various mission phases is a second desired figure of merit. This result is monotonically greater than an estimate based on just the final δV and the initial mass at each stage.

Mission Profiles

Whereas there are many mission architectures,[1,2] allowing a spacecraft to travel from LEO to the moon or Mars, a standard must be applied to allow consistent comparison for this paper. Note that only travel to a destination is discussed here; a return flight is not part of this analysis.

In the lunar scenario, the spacecraft starts in LEO. After its engines are fired for a translunar injection, the craft coasts to the vicinity of the moon and then brakes into a lunar orbit. After a short period of time in lunar orbit, the craft's engines fire again to slow it out of orbit, then continuously to deposit it on the lunar surface.

For lunar-produced fuel, mining equipment (private communication, I. N. Sviatoslovsky) on the surface gathers the necessary feedstock and removes the desired volatiles. After extraction, the volatiles are launched off the surface of the moon to aerocapture into LEO and rendezvous with the Mars craft. The mining equipment is assumed to require only a small crew (four to six) and need no added mass for maintenance. No allowance is made for the mass cost of ferry craft carrying the volatiles from the moon to LEO, but

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the mass added to the 18 t miner is estimated at 25 t for habitat, 10 t for a solar collector, 6 t for a solar radiator, plus 3 t for surface vehicles and other minor units, that is, 62 t in all.

For a Mars mission, the craft leaves LEO on a trans-Mars trajectory (using propellant launched with it from Earth or separately from the moon), approaching the planet and aerocapturing into orbit. After spending a period of time in that orbit, the craft makes a small velocity change using thrusters that brings it into Mars's atmosphere. Both the deorbit burn and any trim maneuvers required as part of aerocapture are roughly estimated as part of this analysis. On the way to the surface, the aeroshell absorbs most of the orbital velocity of the craft. After the deceleration to subsonic speeds, the aeroshell is discarded and parachutes are deployed to slow the craft to a few meters per second. Rocket engines are used for final approach to the landing site and deceleration to the surface.

III. Analysis

To examine the cumulative impact of these rules on four distinct moon and Mars mission scenarios, we start from a common point, namely, LEO. For each case, we keep a constant fraction of the needed fuel mass increment as an estimate of tankage and booster structure mass,[3] that is, $m_{d,k} = (0.1)m_{k-1}\eta_f$. The lunar excursion is based on classic Apollo mission parameters for the mission segments beyond LEO.

For the Mars excursion with fuel from the moon, the fuel mass added is to be that delivered to LEO after a boost to Earth–moon L1 and an aerocapture. As with the other mission segments, if this boost were done with some of the lunar fuel, a slight penalty factor for it would arise to determine the needed fuel mass produced on the lunar surface. Insofar as this mass penalty is modest, and other means than rocketry could be used to launch the fuel from the moon, this added penalty is not reflected in our Table 1 (Refs. 4–6).

For the moon to Mars excursion, the dry tankage and payload mass of 1.4865 kg projected back from Mars is treated as landed payload on the moon. Hence, it is then penalized by the full 9.2623-kg multiplier back to LEO to compute the needed dry mass to be sent up for the lunar segment.

All alternatives use a fixed booster specific impulse[7] of 325 s, except where moon derived H_2 and O_2 fuel is assigned a higher specific impulse of 455 s. Also the Mars examples in Tables 1 and 2 take, from modern designs, a reasonable mass fraction[8] for the final aeroshell at $(0.15)m_1$.

Hence, in Table 1 we show the accumulated mass penalties back to LEO for the various mission profiles: moon, Mars direct, Mars with moon fuel, and moon to Mars. Reading down Table 1, these penalties are grouped into three classes: landing, capture at the target body, and injection into the transfer orbit from LEO. In Table 2. we show a similar comparison of the energy invested to land one kilogram of payload for each of the mission types.

Whereas the aeroshell mass fraction is subject to some variation with design choices, the behavior shown in Table 1 is not substantially altered were this fraction to rise, even up to 0.50. Moreover, varying a second critical parameter as well, Fig. 1 shows the relative mass advantage to a Mars mission over a moon mission, namely, $M_{\mathrm{LEO,moon}}/M_{\mathrm{LEO,Mars}}$, as a function of the aeroshell mass fraction and the specific impulse of the transfer orbit injection only. The relative mass advantage to Mars ranges from 1.26 up to 2.31 over the parameter domain shown there.

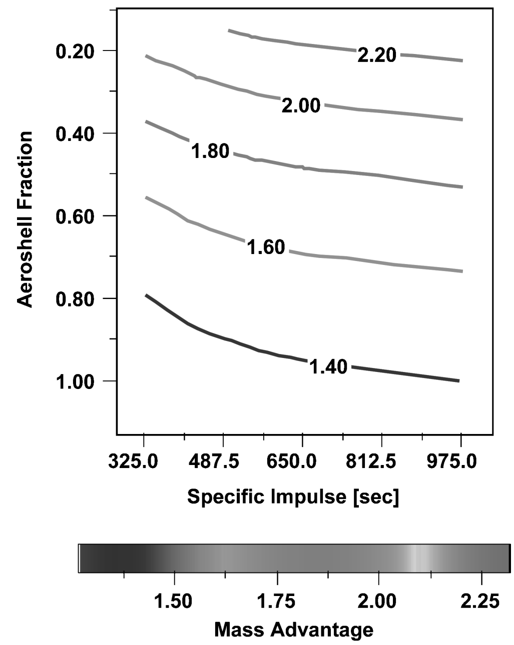

Fig. 1 Relative mass advantage, Mars/moon.

Table 1 Mass breakdown for four missions

Stage	Moon[4]	Mars direct[5,6]	Mars with moon fuel	Mars via moon
Cargo landed	1.0000 kg	1.0000 kg	1.0000 kg	1.0000 kg
δV[a]	2.6840 km/s	0.4000 km/s	0.4000 km/s	0.4000 km/s
Deorbit fuel	2.3213 kg	1.1337 kg	1.1337 kg	1.1337 kg
Tankage	0.1321 kg	0.0134 kg	0.0134 kg	0.0134 kg
Total[b]	2.4534 kg	1.1471 kg	1.1471 kg	1.1471 kg
Aeroshield	0.0000 kg	0.1500 kg	0.1500 kg	0.1500 kg
Total[c]	2.4534 kg	1.2971 kg	1.2971 kg	1.2971 kg
δV[d]	0.8600 km/s	0.1000 km/s	0.1000 km/s	0.1000 km/s
Capture fuel	3.2134 kg	1.3384 kg	1.3384 kg	1.3384 kg
Tankage	0.0760 kg	0.0041 kg	0.0041 kg	0.0041 kg
Total[e]	3.2894 kg	1.3426 kg	1.3426 kg	1.3426 kg
δV[f]	3.1070 km/s	3.6000 km/s	3.6000 km/s	3.2500 km/s
Transit fuel[g]	8.7193 kg	4.1542 kg	3.0084 kg	2.7814 kg
Tankage	0.5430 kg	0.2812 kg	0.1666 kg	0.1439 kg
Final	9.2623 kg	4.4353 kg	1.5091 kg	2.9253 kg[h] 13.7680 kg[i]

[a]To land. [b]Before landing. [c]Before entry. [d]To capture. [e]Before capture. [f]To transfer orbit. [g]Boosted or added. [h]Before injection. [i]Drymass in LEO.

Table 2 Energy cost per landed kilogram by mission type

Stage	Mission specific energy, MJ/kg Moon	Mars direct	Mars with moon fuel
Landing	4.867	0.083	0.083
Capture	0.995	0.007	0.007
Injection	22.585	13.156	11.599
Total energy	28.447	13.246	11.689

IV. Conclusions

From these comparisons, it is clear that Mars is a more attractive target based on either mass or energy cost per unit of delivered payload. Our analysis is most directly relevant to simple cargo missions, but certainly crewed missions are subject to the same fundamental conclusions with the proviso that some of the payload must include life support expendable mass. Clearly longer duration crewed Mars missions must be significantly more massive than lunar excursions, but the cost per payload mass is uniformly more favorable for a Mars flight. Moreover, if the trans Mars injection (and trans lunar injection) burns are done with a higher specific impulse stage, then the mass ratio in favor of Mars direct rises to about 2.31 because the fuel mass is used more efficiently for a larger velocity increment.

With fuel from the moon, the mass advantage to Mars rises to a rather compelling value of 6.14. Yet, if the use of lunar fuel is to be considered, then the equipment to mine it must be boosted from LEO and soft landed on the moon. If the mass required in LEO to boost the mining package out to the the moon is larger than the fuel mass one can ever hope to recover from the moon, then clearly the better choice is still bringing all of the fuel mass directly to LEO. Should the fuel mass delivered from the moon take too long to reach parity with the original boost mass, the miner may require even more mass for maintenance before its mass debt is ever paid.

If we examine one point design (private communication, Sviatoslovsky) for such a miner and its supporting equipment, the mining package runs to a total mass of about 62 t, with 18 t being the miner itself. Covering about 1 km^2/year, such a miner would perhaps yield about 600 t/year in gas to be fractionated cryogenically. Of that mass (private communication, G. L. Kulcinski), only perhaps 0.18 of it is H_2O, and 0.33–0.38 of it is H_2, so that the useful fuel mass per year comes to only about 108 t/year in directly extracted H_2O. With the further 198 t/year of hydrogen available, one can imagine its use either as a simple fuel or as equivalent to 1782 t/year water mass, once additional oxygen is entrained from the reduction of lunar ilmenite with that hydrogen.

The LEO mass cost to throw 62 t of mining equipment is then 574.3 t, which could have been fuel pushed up directly. Hence, the lowest fuel production in direct H_2O requires more than 5 years of good, repair-free mining to pay the mass trade. If we make the generous assumptions of 1) a viable deposit of ilmenite and 2) only a few tonnes mass for a reduction plant, the mass trade payoff time comes down to 0.3 years.

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